Effect of numerical aperture on the spectral splitting feature near phase singularities of focused waves

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We demonstrate that because of the depolarization effect associated with a high-numerical-aperture lens, the recently predicted spectral splitting phenomenon near phase singularities of focused waves [G. Gbur, T. D. Visser, and E. Wolf, *Phys. Rev. Lett.* **88**, 013901 (2002)] disappears when the numerical aperture is higher than critical values that are different between the incident polarization direction and the axial direction.
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In a recent publication,[1] the spectral behavior of a focused wave near the points at which the field amplitude has the zero value has been investigated. It has been shown that when the incident field is polychromatic rather than monochromatic, but spatially fully coherent, the spectrum at the zero-intensity points, called phase singularities,[1] exhibits the anomalous behavior that causes the splitting of the spectrum. The authors have considered an incident field consisting of a narrow spectral line centred at a frequency ω_0, focused by an aperture. In their treatment, the field in the focal region is calculated using a scalar diffraction theory given by Lommel functions.[2] This theory is valid in the case when the diffraction aperture has a small maximum angle of convergence, which is called the paraxial approximation. Such a diffraction system corresponds to a low-numerical-aperture (NA) lens. However, when the NA becomes large, the focusing process involves depolarization. In other words, a linearly polarized incident beam E_x exhibits two extra components, one in the orthogonal direction, E_y, and the other one in the longitudinal direction, E_z. Consequently, the spectral splitting phenomenon predicted recently[1] may not necessarily appear in the focal region of a high-NA lens. The aim of this letter is to demonstrate the dependence of the spectrum at the phase singularities on the NA.

Before we demonstrate the dependence of the anomalous spectral behavior on the NA of a focusing lens, it is necessary to point out that illumination by a polychromatic and spatially coherent light beam,[1] corresponding to an incoherent superposition of the spectral components, is equivalent to ultrashort-pulsed laser illumination. An ultrashort-pulsed wave consists of many frequency components or modes that are coherently superposed together, which is called the mode-locking technique.[3] It has been demonstrated[4,5] that the spectral distribution in the focal region is the same as that given by Eq. (4) in Ref. 1. Therefore, the situation considered in Ref. 1 corresponds to an ultrashort-pulsed beam of 140 fs.

An ultrashort-pulsed beam has been increasingly used in different imaging techniques, such as time-resolved optical microscopy and nonlinear optical microscopy.[6] These imaging techniques implement microscope objectives of high NA to form a submicrometer focal spot. Therefore, it is important to discuss the effect of the reported anomalous behavior of spectra near phase singularities of focused waves,[1] when the NA is large. The expression of the three electric field components E_x, E_y, and E_z in the focal region of a high NA lens that satisfies the sine condition[2] has been derived by

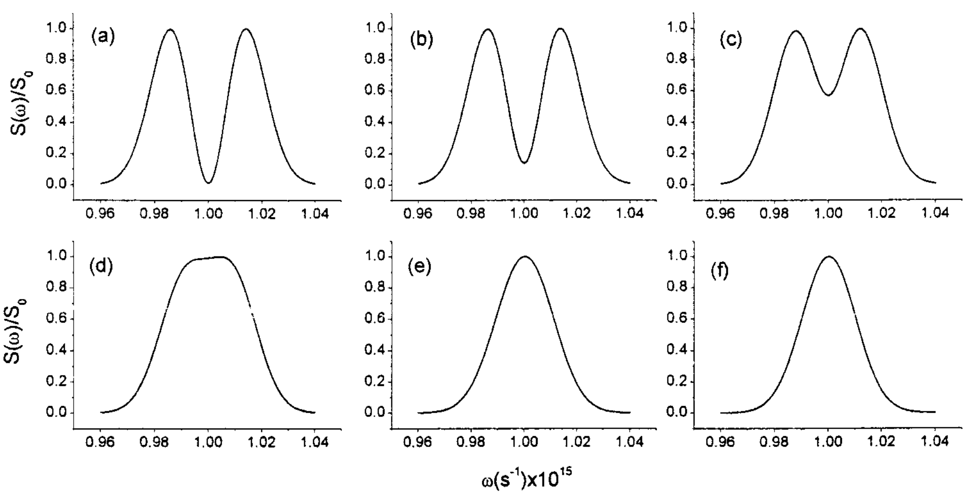

FIG. 1. The normalized spectrum $S[u_1(\omega_0),\omega]/S_0$ at the first axial zero-intensity point ($u_1=4\pi$) of the central frequency component ω_0 for different values of NA: (a) NA=0.025, (b) NA=0.1, (c) NA=0.3, (d) NA=0.4, (e) NA=0.6, (f) NA=0.9.

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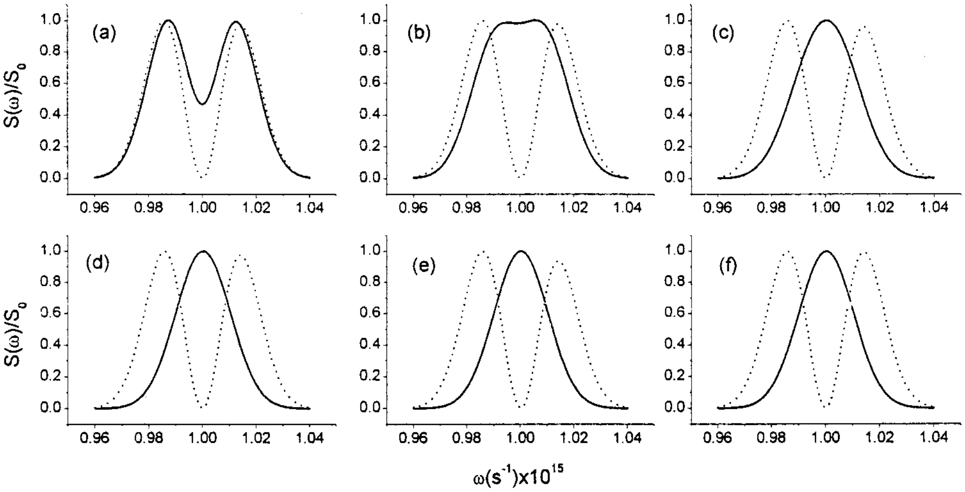

FIG. 2. The normalized spectrum $S(\omega)/S_0$ at the first x and y zero-intensity points of the central frequency component ω_0 for different values of NA: (a) NA=0.025, (b) NA=0.05, (c) NA=0.1, (d) NA=0.3, (e) NA=0.6, (f) NA=0.9. Full lines show the variations in the x direction, while dotted lines represent those in the y direction.

Richards and Wolf.[7] The strength of the longitudinal component E_z becomes large when the NA approaches unity. Consequently, the minimum-intensity point in the focal region may have a nonzero value rather than the zero value as predicted by the scalar theory under the paraxial approximation.[1,2]

According to the results presented in Ref. 1, the spectrum of the light intensity at the minimum intensity points is split into two lines because the spectral intensity at the frequency ω_0 is zero at those points. Using the vectorial diffraction theory,[7] we have shown that the intensity at those points is not zero,[8] and thus can reveal the dependence of the spectral splitting on the NA of a focusing lens.

The anomalous behavior of the spectral intensity, calculated at the axial zero-intensity points under the paraxial approximation, occurs because of the condition that the spectral intensity at frequency ω_0 has a zero value when $u_n(\omega_0) = 4\pi n$ is valid for low NA lenses. These zero-intensity points disappear for lenses with high NA as a result of the contribution of the depolarized longitudinal component E_z.[8]

Figure 1 shows the normalized spectrum at the first axial minimum intensity point of the central frequency component ω_0 for different values of the NA. For a low-NA lens [Fig. 1(a)], the normalized spectrum is split into two lines of equal intensity. This result corresponds to the one obtained by Ref. 1, as, for this case, the paraxial approximation applies. However, when the maximum angle of convergence of the lens increases, the spectrum at the first axial minimum intensity point undergoes a noticeable change. When the NA increases further [Figs. 1(b) and 1(c)], the spectrum is still split into two lines, but with a shallow dip in the center. The slight asymmetric feature is caused by the $1/\lambda$ factor in Eq. (4) of Ref. 1. When the NA is larger than 0.4 [Fig. 1(d)], the dip in the spectrum disappears and the spectrum does not split. In the case of a high-NA lens [Figs. 1(e) and 1(f)], the spectrum distribution is almost the same as the input spectral distribution.

Another consequence of the depolarization effect is the elongation of the focal spot along the incident polarization direction.[5,7] While the first zero-intensity point still exists in the y direction,[8] the first minimum-intensity point shows a nonzero value in the x direction that is the incident polarization direction. As a result, the spectral splitting in the y direction is independent of the NA value, while the spectral behavior in the incident polarization direction is NA dependent (Fig. 2). For a low-NA lens [Fig. 2(a)], the spectrum is split in the x direction; however the spectrum already shows a nonzero dip at the central frequency component ω_0. The spectral splitting in the x direction disappears at NA=0.05 [Fig. 2(b)], and the spectrum approaches that of the incident beam quickly as the NA value increases [Figs. 2(c)–2(f)].

In conclusion, because of the depolarization caused by a high-NA lens, the spectral splitting feature near the minimum-intensity points under polychromatic illumination exists neither in the incident polarization direction nor in the axial direction when the NA of a lens larger than critical values. Such a critical NA value in the incident polarization direction is as small as 0.05. The knowledge of the spectral dependence on the NA is important when one uses an ultrashort-pulsed beam to perform time-resolved or nonlinear optical microscopy and spectroscopy.

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